JASA REFLECTIONS

The *Reflections* series takes a look back on historical articles from *The Journal of the Acoustical Society of America* that have had a significant impact on the science and practice of acoustics.

Testing the theoreticians

Article: The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian
Author: Eric I. Thorsos
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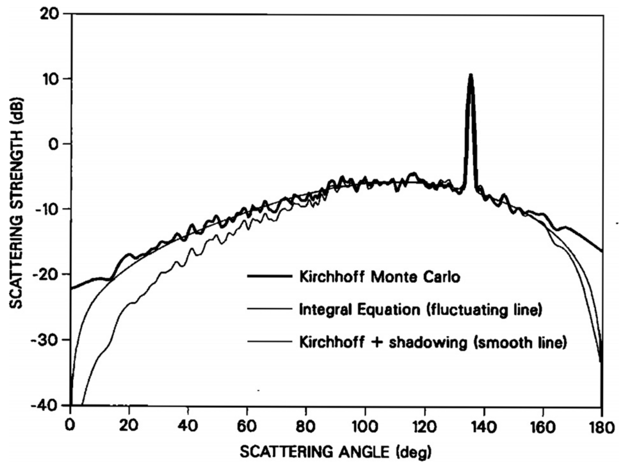
HISTORICAL SETTING

By the 1970s, there were several competing approximate models for wave scattering from rough surfaces. The most important two methods were the tangent plane approximation (known as the Kirchhoff approximation), and the small roughness perturbation approximation. Although these models were widely used to model scattering by natural surfaces in both the acoustics and radar communities, they had not been rigorously verified by comparison to a reference solution. Many researchers were using the Helmholtz-Kirchhoff integral technique, an exact solution for arbitrary rough surface scattering, to provide the reference solution, but the results were corrupted by the effects of surface truncation.[1–3] Eric Thorsos was the first to find an accurate implementation of the integral equation method for wave scattering from rough surfaces, providing the much-needed reference solution for approximate models.[4]



Comparison of the scattering cross section versus grazing angle calculated using three different methods. [Reprinted with permission from Thorsos, J. Acoust. Soc. Am. Vol 83, 78–92 (1988). Copyright 1988 Acoustical Society of America (Ref. 4).]

ARTICLE CONTRIBUTIONS

An exact, numerical prediction of the scattered field from an infinitely long surface requires truncation. Previous, theoretical models used monochromatic plane waves of infinite extent as the incident field, which are unrealizable experimentally and numerically. Thorsos's innovation was to use a tapered beam of incident sound that decayed rapidly, such that the effects of truncation were negligible. A simple Gaussian-function taper results in an incident field that does not satisfy the Helmholtz equation, and thus produces different results when different formulations of the integral equation are used. Thorsos developed a modified Gaussian beam, which included a correction term that brought these integral equations into better accord, with accuracy on the order of $(kg\sin\theta)^{-2}$, where k is the acoustic wavenumber, g is the physical extent (width) of the beam, and θ is the grazing angle. This expression for the degree of accuracy allows users of the technique to directly estimate the faithfulness of the solution. Using this numerically robust technique allowed Thorsos to perform a systematic study of the Kirchhoff approximation, which constitutes the bulk of the 1988 paper.[1]

In Ref. 4, Thorsos introduced this modified formulation and analyzed the validity of the Kirchhoff approximation using a surface with Gaussian height statistics and a Gaussian spectrum. Thorsos found that the Kirchhoff approximation was accurate for moderate grazing angles, when the root mean square (rms) slope angle (i.e., the arctangent of the rms slope) of the surface was small compared to the grazing angle of the incident beam, and the correlation length was larger than the wavelength. When these conditions were not satisfied, the Kirchhoff approximation is not accurate because shadowing and multiple scattering played an important role. One of the many comparisons between the integral equation technique and the Kirchhoff approximation is shown in Fig. 1 reproduced from Ref. 4. This figure shows the scattering cross section per unit angle per unit length as a function of scattered grazing angle (i.e., the direction of the outgoing wave) computed using the different models. When the model and integral equation lines are close, the model is accurate. When the two lines significantly depart (as they do for small grazing angles in this figure), then the Kirchhoff approximation is not accurate.

IMPACT OF THE ARTICLE

Although this paper was published in JASA, its impact has extended far beyond acoustics. Theoretical scattering models are required for remote sensing in the radar, optics, ultrasound, and nanometrology communities. This paper has had an influence on all of these communities, as seen by the composition of the 1251 citations it has received to date. Several books are listed as highly cited citations to this article, including monographs on wave scattering theory,[5] surface metrology,[6] physical optics,[7] and microwave remote sensing.[8] This paper is also referenced in two highly cited reviews of the literature.[9,10]

Thorsos (and colleagues) used this technique to study other approximations[11–13] and scattering by the ocean surface.[14] This technique has been used to study different boundary conditions, such as a fluid-fluid interfaces,[15,16] and layered seafloors.[17,18] Complementary techniques, such as the finite element method (also using the extended Gaussian Beam) have been used to investigate the validity of models for scattering from fluid and elastic rough interfaces.[19] Combined with Fourier synthesis, Thorsos's method has also been used to investigate scattering of broadband signals from rough interfaces, which is more directly comparable to experiments.[17,20] A thorough study of the incident beam has been performed by researchers in the electromagnetic scattering community,[21] including its effects in both kinds of integral equations. Thorsos' 1988 paper provided a solution to a long-standing problem in rough surface scattering that provided the foundation for work in many areas of physical science.

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REFERENCES

[1]R. R. Lentz, "A numerical study of electromagnetic scattering from ocean-like surfaces," Radio Sci. 9, 1139–1146 (1974), https://doi.org/10.1029/RS009i012p01139.

[2]R. Axline and A. Fung, "Numerical computation of scattering from a perfectly conducting random surface," IEEE Trans. Antennas Propag. 26, 482–488 (1978).

[3]R. Axline and A. Fung, "Corrections to 'Numerical computation of scattering from a perfectly conducting random surface,'" IEEE Trans Antennas Propag. 28, 949–949(1980).

[4]E. I. Thorsos, "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," J. Acoust. Soc. Am. 83, 78–92 (1988).

[5]A. G. Voronovich, *Wave Scattering from Rough Surfaces*, 2nd ed. (Springer, Berlin, 1999).

[6]D. J. Whitehouse, *Handbook of Surface and Nanometrology* (Taylor & Francis, New York, 2002).

[7]M. Nieto-Versperinas, *Scattering and Diffraction in Physical Optics*, 2nd ed. (World Scientific, Hackensack, NJ, 2006).

[8]D. P. Winebrenner, J. Bredow, A. K. Fung, M. R. Drinkwater, S. Nghiem, A. J. Gow, D. K. Perovich, T. C. Grenfell, H. C. Han, J. A. Kong, J. K. Lee, S. Mudaliar, R. G. Onstott, L. Tsang, and R. D. West, "Microwave sea ice signature modeling," in *Microwave Remote Sensing of Sea Ice*, edited by F. D. Carsey (Americal Geophysical Union, Washington, DC, 1992), Chap. 8, pp. 137–175.

[9]T. M. Elfouhaily and C-A Guèrin, "A critical survey of approximate scattering wave theories from random rough surfaces," Waves Rand. Media 14, R1–R40 (2004).

[10]K. F. Warnick and W. C. Chew, "Numerical simulation methods for rough surface scattering," Waves Rand. Media 11, R1–R30 (2001).

[11]E. I. Thorsos and D. R. Jackson, "The validity of the perturbation approximation for rough surface scattering using a Gaussian roughness spectrum," J. Acoust. Soc. Am 86 261–277 (1989).

[12]S. L. Broschat, E. I. Thorsos, and A. Ishimaru, "The phase perturbation technique vs. an exact numerical method for random rough surface scattering," J. Elecromag. Waves Appl. 3, 237–256 (1989).

[13]S. L. Broschat and E. I. Thorsos, "An investigation of the small slope approximation for scattering from rough surfaces. Part II. Numerical studies," J. Acoust. Soc. Am. 101, 2615–2625 (1997).

[14]E. I. Thorsos, "Acoustic scattering from a 'Pierson-Moskowitz' sea surface," J. Acoust. Soc. Am. 88, 335–3349 (1990).

[15]E. I. Thorsos, D. R. Jackson, and K. L. Williams, "Modeling of subcritical penetration into sediments due to interface roughness," J. Acoust. Soc. Am. 107, 263–277 (2000).

[16]A. A. Maradudin, T. Michel, A. R. McGurn, and E. R. Mendez, "Enhanced backscattering of light from a random grating," Ann. Phys. 203, 255–307 (2000).

[17]D. J. Tang and B. T. Hefner, "Modeling interface roughness scattering in a layered seabed for normal-incident chirp sonar signals," J. Acoust. Soc. Am. 131, EL302–EL308 (2012).

[18]D. R. Olson and D. Jackson, "Scattering from layered seafloors: Comparison between theory and integral equations," J. Acoust. Soc. Am. 148, 2086–2095 (2020).

[19]M. J. Isakson and N. P. Chotiros, "Finite element modeling of acoustic scattering from fluid and elastic rough interfaces," IEEE J. Ocean. Eng. 40, 475–484 (2015).

[20]D. R. Olson and A. P. Lyons, "Resolution dependence of rough surface scattering using a power law roughness spectrum," J. Acoust. Soc. Am. 149, 28–48 (2021).

[21]J. Toporkov, R. S. Awadallah, and G. S. Brown, "Issues related to the use of a Gaussian-like incident field for low-grazing-angle scattering," J. Opt. Soc. Am. A 16, 176–187 (1999).

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